RESEARCH ARTICLE OPEN ACCESS

Medical Diagnostic Reasoning Using Extended Hausdorff Distance for Intuitionistic Fuzzy Sets

P. Thiruveni*, B. Mohamed Harif**, A. Mohamed Ali***
*Department of Mathematics, Rajah Serofji Government College. Thanjavur.(T.N) India.
**Assistant professor of Mathematics, Rajah Serofji Government College. Thanjavur.(T.N) India.
***Department of Mathematics, Jamal Mohamed College, Trichirappalli .(T.N) India

ABSTRACT
In this paper a new method to find a medical diagnostic reasoning with the help of extended huasdroff distance for intuitonistic fuzzy sets.
***Keywords*:** Intuitionistic fuzzy set, Extended Hausdroff distance, Normalized Hamming distance.
2010 Mathematics Subject Classification: 03E72, 03F55

I. INTRODUCTION

Fuzzy set was proposed by Zadeh in 1965 as a frame work to encounter uncertainly, vagueness and partial truth, represents a degree of membership for each member of the universe of discourse to a subset of it. Intuitionistic fuzzy set was proposed by Attanassov, [1], [2] in 1986 which looks more accurate to uncertainty quantification and provided the opportunity to precisely model the problem based on the existing knowledge and observations. The Intuitionistic fuzzy set theory has been studied and applied in different areas.

In this paper intuitionistic fuzzy sets as a tool for a more human consistent reasoning under imperfectly defined facts and imprecise knowledge, with an application in supporting medical diagnosis. More specifically, we have set of data, i.e. a description of a set of symptoms S, and a set of diagnoses D, describe a state of a patient knowing results of his/her medical tests are discussed. Using the concept of an intuitionistic fuzzy set that makes it possible to express many new aspects of imperfect information. For instance, in many cases information obtained cannot be classified due to lack of knowldge, disciminating power of measureing tools, etc. In such a case the use of a degree of membership and nonmebership can be an adequate knowledge representation solution.

The hausdroff distances play a important role in practical application, notably in image matching, image analysis, motion tracking, visual navigation of robots, computer-assisted surgery and so on.

Moreover, with the increased volume of information available to physicians from new medical technologies, the process of classifying different sets of symptoms under a single name of disease and determining the appropriate therapeutic actions becomes increasingly difficult. Recently, there are varieties of models of medical diagnosis under the general framework of fuzzy sets theory involving Intuitionistic fuzzy set to deal with different complicating aspects of medical diagnosis.

II. PRELIMINARIES AND DEFINITION

A Fuzzy set A is defined by
$A = \{(x, \mu_A (x) \} : x \in A, \mu_A (x) \in [0,1] \}$

In the pair $(x, \mu_A (x))$, the 1^{st} element x belong to the classical set A, the second element $\mu_A (x)$ belong to the interval [0,1] called membership function.

The fuzzy set of expansive Fruits can be described as {(Apple, 1) Straw Berry, 1) (Cherry, 0.8) (Orange, 0.4) (Mango, 0.3)}

The fuzzy set A is included in the fuzzy set B denoted by $A \leq B$ if for every $x \in X$ then $\mu_A (x) \leq \mu_B (x)$. Then A is called a subset of B. Where X is the universal crisp set.

Fuzzy set A & B are called equal if $\mu_A (x) = \mu_B (x)$ for every element $x \in X$.This is denoted by A = B.

Let A and B be two fuzzy sets defined on the universe of discourse X

$\mu_{AuB} (x) = \mu_A(x) \vee \mu_B(x)$

ie,. $\mu_{AuB} (x) = \text{Max} \{\mu_A(x) , \mu_B(x)\}$

The intersection of fuzzy set A and B is A n B such that
$\mu_{AnB} (x) = \min \{ \mu_A (x) , \mu_B (x) \}$.
Here μ_{AnB}, $\mu_{A,}$ μ_B are the membership grade of an element x.

The complement of the fuzzy set with respect to the universal set X by $\bar{A}$ and is defined it by
$\mu_{\bar{A}} (x) = 1 - \mu_A (x)$ for every $x \in X$.

The support of a fuzzy set A is the set of all points x in X such that
$\mu_A (x) > 0$. That is Support (A) = $\{ x / \mu_A (x)>0 \}$.

Let A be a fuzzy set defined on X and any number α Є [0,1], the α- cut is the crisp set denoted by $^{\alpha}A$ and is defined by

$$^{\alpha}A = \{x : A(x) \geq \alpha\}$$

If the fuzzy set A and B are not equal if $\mu_A(x) \neq \mu_B(x)$ for at least one x Є X.This is denoted by $A \neq B$.

Fuzzy set A is called proper subset of fuzzy set B when A⊂B and the two sets are not equal.

ie, $\mu_A(x) \leq \mu_B(x)$ for every x Є X, and $\mu_A(x) < \mu_B(x)$ for at least one x Є X.

Intuitionistic fuzzy set was introduced first time by Atanassov, which is a generalization of an ordinary Zadeh fuzzy set. Let X be a fixed set. An intuitionistic fuzzy set A in X is an object having the form $A = \{(x, \mu_A(x), v_A(x)) | x \in X\}$ (1)

where the functions $\mu_A(x), v_A(x) : X \to [0,1]$ are the degree of membership and the degree of non-membership of the element $x \in X$ to A, respectively;moreover, $0 \leq \mu_A(x) + v_A(x) \leq 1$ must hold.

Obviously, each fuzzy set may be represented by the following intuitionistic fuzzy set

$$A = \{(x, \mu_A(x), 1 - \mu_A(x) | x \in X\} \quad (2)$$

In addition to that, we also include the hesitation margin, $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$

(i.e) $A = \left\{\left(x, \mu_A(x), v_A(x), \pi_A(x)\right) / x \in X\right\}$ (3)

III. DISTANCE BETWEEN INTUITIONISTIC FUZZY SET

In Szmidt and Kacprzyk [3], [4], it is shown why in the calculation of distances between the intuitionistic fuzzy sets one should use all three terms describing them. Let A and B be two intuitionistic fuzzy set in $= \{x_1, x_2, \dots, x_n\}$. Then the distance between A and B while using the three term representation (Szmidt and Kacprzyk) may be as follows.

The Hamming distance:

$$d_{IFS}(A,B) = \frac{1}{2}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \quad (4)$$

The Euclidean distance:

$$e_{IFS}(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left(\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(v_A(x_i) - v_B(x_i)\right)^2 + (\pi_A(x_i) - \pi_B(x_i))^2\right)} \quad (5)$$

The normalized Hamming distance:

$$l'_{IFS}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}\left(|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|\right) \quad (6)$$

The normalized Euclidean distance:

$$q'_{IFS}(A,B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left(\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(v_A(x_i) - v_B(x_i)\right)^2 + (\pi_A(x_i) - \pi_B(x_i))^2\right)} \quad (7)$$

IV. THE HAUSDROFF DISTANCE

Given two intervals $U = [u_1, u_2]$ and $V = [v_1, v_2]$ of the hausdroff metric is defined [5]

$$d_H(U,V) = \max\{|u_1 - v_1|, |u_2 - v_2|\} \quad (8)$$

The hausdroff metric applied to two intuitionistic fuzzy sets, $A(x) = [\mu_A(x), 1 - v_A(x)]$ and $B(x) = [\mu_B(x), 1 - v_B(x)]$, given the following:

$$d_H(A(x), B(x)) = \max\{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\} \quad (9)$$

The following two term representation hausdroff distances between intuitionistic fuzzy sets have been proposed [5]:

The Hamming distance :

$$d_H(A,B) = \sum_{i=1}^{n}\max\left\{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\right\} \quad (10)$$

The normalized Hamming distance:

$$l_H(A,B) = \frac{1}{n}\sum_{i=1}^{n}\max\{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\} \quad (11)$$

The Euclidean distance :

$$e_H(A,B) = \sqrt{\sum_{i=1}^{n} max\left(\left(\mu_A(x_i) - \mu_B(x_i)\right)^2, (v_A(x_i) - v_B(x_i))^2\right)} \quad (12)$$

The normalized Euclidean distance :

$$q_H(A,B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} max\left(\left(\mu_A(x_i) - \mu_B(x_i)\right)^2, (v_A(x_i) - v_B(x_i))^2\right)} \quad (13)$$

Let $A = \{< x_i, \mu_A(x_i), v_A(x_i), \pi_A(x_i) > / x_i \in X\}$ and $B = \{< x_i, \mu_B(x_i), v_B(x_i), \pi_B(x_i) > / x_i \in X\}$ be the two intuitionistic fuzzy sets of the universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, then

The Hamming distance:

$$d_{EH}(A,B) = \sum_{i=1}^{n}\max\left\{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|, |\pi_A(x_i) - \pi_B(x_i)|\right\} \quad (14)$$

The normalized Hamming distance $l_{EH}(A,B)$

$$l_{EH}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \max \begin{Bmatrix} |\mu_A(x_i) - \mu_B(x_i)|, \\ |v_A(x_i) - v_B(x_i)|, |\pi_A(x_i) - \pi_B(x_i)| \end{Bmatrix} \quad (15)$$

The Euclidean distance $e_{EH}(A,B)$

$$e_{EH}(A,B) = \sqrt{\sum_{i=1}^{n} \max \begin{pmatrix} (\mu_A(x_i) - \mu_B(x_i))^2, \\ (v_A(x_i) - v_B(x_i))^2, \\ (\pi_A(x_i) - \pi_B(x_i))^2 \end{pmatrix}} \quad (16)$$

The normalized Euclidean distance

$$q_{EH}(A,B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \max \begin{pmatrix} (\mu_A(x_i) - \mu_B(x_i))^2, (v_A(x_i) - v_B(x_i))^2 \\ (\pi_A(x_i) - \pi_B(x_i))^2 \end{pmatrix}} \quad (17)$$

V. MEDICAL DIAGNOSTIC REASONING VIA EXTENDED HAUSDROFF DISTANCE FOR INTUITIONISTIC FUZZY SETS

In real world, we frequently deal with vague or imprecise information. Information available is sometimes vague, sometimes inexact or sometimes insufficient. Out of several higher order fuzzy sets, intuitionistic fuzzy sets (IFS) have been found to be highly useful to deal with vagueness. There are situations where due to insufficiency in the information available, the evaluation of membership values is not possible up to our satisfaction. Due to the some reason, evaluation of non-membership values is not also always possible and consequently there remains a part in-deterministic on which hesitation survives. Certainly fuzzy sets theory is not appropriate to deal with such problem; rather IFS theory is more suitable. Out of several generalizations of fuzzy set theory for various objectives, the notion introduced by Atanassov in defining intuitionistic fuzzy sets is interesting and useful. Fuzzy sets are intuitionistic fuzzy sets but the converse is not necessarily true. In fact there are situations where IFS theory is more appropriate to deal with [6]. Besides, it has been cultured in [7] that vague sets [8] are nothing but IFS.

Let there be four patients Kumar, Mani, Ramraj, and Kannan i.e., P = {Kumar, Mani, Ramraj, Kannan} and the set of symptoms S = {temperature, headache, stomach pain, cough and chest - pain}. Let the set of Diagnosis be D = {Viral Fever, Malaria, Typhoid, Stomach Problem, Chest Problem}. The data are given in Table-1 each symptom is described by: membership μ, non-membership v, hesition margin π. The symptoms are given in Table 2 – as before, we need all three parameters (μ,v,π) to describe each symptom. We seek a diagnosis for each patient P_i, i= 1, . . . , 4. I proposed to solve the problem in the following way:

i. to calculate for each patient *Pi* a distance (I used the Extended Hausdroff – normalized hamming distance) of his symptoms (Table 2) from a set of symptoms *Sj* , *j* = 1, . . . , 5 characteristic for each diagnosis *dk*, *k* = 1, . . . , 5 (Table 1)

ii. to determine the lowest distance which points out to a proper diagnosis.

The Extended haudroff - normalised Hamming distance for all symptoms of patient *i*-th from diagnosis *k* is

$$l_{EH}(s(p_i),d_k) = \frac{1}{5}\sum_{j=1}^{5} max\{|\mu_j(p_i) - \mu_j(d_k)|, |v_j(p_i) - v_j(d_k)|, |\pi_j(p_i) - \pi_j(d_k)|\} \quad \textbf{(18)}$$

Table 1: Symptoms characteristic for the diagnoses considered

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Temperature	(0.4,0.0,0.6)	(0.0,0.7,0.3)	(0.3,0.3,0.4)	(0.1,0.7,0.2)	(0.1,0.8,0.1)
Headache	(0.3,0.5,0.2)	(0.2,0.6,0.2)	(0.6,0.1,0.3)	(0.2,0.4,0.4)	(0.0,0.2,0.8)
Stomach pain	(0.1,0.7,0.2)	(0.0,0.9,0.1)	(0.2,0.7,0.1)	(0.8,0.0,0.2)	(0.2,0.8,0.0)
Cough	(0.4,0.3,0.3)	(0.7,0.0,0.3)	(0.2,0.6,0.2)	(0.2,0.7,0.1)	(0.2,0.8,0.0)
Chest pain	(0.1,0.7,0.2)	(0.1,0.8,0.1)	(0.1,0.9,0.0)	(0.2,0.7,0.1)	(0.8,0.1,0.1)

Table 2: Symptoms characteristic for the patients considered

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Kumar	(0.0,0.8,0.2)	(0.4,0.4,0.2)	(0.6,0.1,0.3)	(0.1,0.7,0.2)	(0.1,0.8,0.1)
Mani	(0.8,0.1,0.1)	(0.8,0.1,0.1)	(0.0,0.6,0.4)	(0.2,0.7,0.1)	(0.0,0.5,0.5)
Ramraj	(0.8,0.2,0.0)	(0.6,0.,0.1)	(0.2,0.7,0.1)	(0.8,0.0,0.2)	(0.2,0.8,0.0)
Kannan	(0.4,0.3,0.3)	(0.7,0.0,0.3)	(0.2,0.6,0.2)	(0.2,0.7,0.1)	(0.2,0.8,0.0)

Table 3 : The Extended Hausdroff – Normalized Hamming distances for each patient from the considered set of possible diagnoses

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Kumar	0.40	0.36	0.32	**0.14**	0.42
Mani	0.38	0.58	**0.32**	0.50	0.56
Ramraj	0.30	0.36	**0.28**	0.54	0.56
Kannan	**0.28**	0.40	0.38	0.44	0.54

VI. CONCLUSION

From the above table, If the doctor agrees, then Kumar faces Stomach problem, Mani and Ramraj suffiers from Typhoid and Kannan suffers from Viral fever.

REFERENCES

[1]. Attanasov K.(1986) *Intuitionistic fuzzy sets, Fuzzy sets and Systems*, 20., 87-96.

[2]. Attanasov K(1999) *Intuitionistic fuzzy sets: Theory and Applications*. Physica-Verlag, Heidelberg and New York.

[3]. Eulalia Szmidt and Janusz Kacprzyk, *Intuitionistic fuzzy sets – two and three term representation in the context of a Hausdroff distance,* Acta Universitatis Matthiae Bell11, series Mathematics 19 (2011) 53-62.

[4]. Szmidt.E and Kacprzyk J (2000) *Distance between intuitionistic fuzzy sets.* Fuzzy sets and systems, 114(3), 505-518.

[5]. Grzegorzewski, *Distances between intuitionistic fuzzy sets and / or interval-valued fuzzy sets based on the hausdroff metric.* Fuzzy Sets and Systems, 48:319-328,2004.

[6]. R. Biswas, *On fuzzy sets and intuitionistic fuzzy sets,* NIFS, 3 (1997), 3-11.

[7]. H. Bustince, P. Burillo, *Vague sets are intuitionistic fuzzy sets,* Fuzzy Sets and Systems, 79 (1996), 403-405.

[8]. W.L. Gau, D.J. Buehrer, Vague sets, IEEE *Trans. Systems Man Cybernet*, 23 (2), (1993), 610-614.